[12-04-25-T11]

Partial fraction decomposition

■ **This is a technique to rewrite a fraction as a sum of fractions. This document treats only one of several cases - the case in which the fraction's denominator can be factored into a product of distinct (no repeated) linear factors. You will learn more about this technique next year.**

EXAMPLE 1.

$$\frac{5}{k^2+5k+6} = \frac{5}{(k+2)(k+3)} = \frac{A}{k+2} + \frac{B}{k+3}$$

$\Longleftrightarrow 5 = A(k+3) + B(k+2)$

Since this is an identity, it is true for all numbers *A, B.* So, it is true for $k = -3$, $k = -2$. Then,

$5 = A(-3+3) + B(-3+2) = -B \Longleftrightarrow B = -5$, and

$5 = A(-2+3) + B(-2+2) = A \Longleftrightarrow A = 5$.

Therefore,

$$\frac{5}{k^2+5k+6} = \frac{5}{k+2} - \frac{5}{k+3}.$$

EXAMPLE 2.

$$\frac{k+4}{k^2+5k+6} = \frac{k+4}{(k+2)(k+3)} = \frac{A}{k+2} + \frac{B}{k+3}$$

$\Longleftrightarrow k + 4 = A(k+3) + B(k+2)$

Since this is an identity, it is true for all numbers *A, B.* So, it is true for $k = -3$, $k = -2$. Then,

$-3 + 4 = A(-3+3) + B(-3+2) = -B \Longleftrightarrow B = -1$, and

$-2 + 4 = A(-2+3) + B(-2+2) = A \Longleftrightarrow A = 2$.

Therefore,

$$\frac{k+4}{k^2+5k+6} = \frac{2}{k+2} - \frac{1}{k+3}.$$

■ Problems.

Rewrite each of the following as a sum or difference of fractions.

[1] $\frac{7}{(x^2-3x-10)}$

[2] $\frac{1}{(2x^2-2)}$

[3] $\frac{7}{(x^2+3x-10)}$

[4] $\frac{3x-1}{(x^2+2x-8)}$

[5] $\frac{5x+3}{(x^2-2x-15)}$

■ Answers.

[1] $\frac{1}{x-5} - \frac{1}{x+2}$

[2] $\frac{1}{4(x-1)} - \frac{1}{4(x+1)}$

[3] $\frac{1}{x-2} - \frac{1}{x+5}$

[4] $\frac{13}{6(x+4)} + \frac{5}{6(x-2)}$

[5] $\frac{3}{2(x+3)} + \frac{7}{2(x-5)}$